Daily rainfall interpolation models obtained by means of genetic programming

Modelos de interpolación de precipitación diaria obtenidos a partir de programación genética

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Abstract

The evolutionary computing algorithm of genetic programming (GP) was applied to obtain mathematical daily rainfall interpolation models in one climatologic station, using the measured data in nearby stations in Cutzamala River basin in Mexico. The obtained models take into account both the geographical coordinates of the climatologic station and also the elevation; the answer of these models was compared against those obtained by means of multiple linear regression, giving genetic programming models a better performance with respect to the multiple linear regression. Isohyets maps were then obtained to compare the spatial shapes between measured and calculated rainfall data in Cutzamala River Basin for a maximum historic storm recorded in year 2006, showing an adequate agreement of the results in case of rainfalls greater than 23 mm. Genetic programming represents a useful practical tool for approaching mathematical models of variables applied in engineering problems and new models could be obtained in several basins by applying these algorithms.

----------*Keywords:* daily rainfall, genetic programming, interpolation models, isohyet, geographic coordinates

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Resumen

Se aplicó el algoritmo de cómputo evolutivo de programación genética (PG) para obtener modelos matemáticos de interpolación de precipitación diaria en una estación climatológica, utilizando datos medidos en las estaciones cercanas a la cuenca del Río Cutzamala en México. Los modelos obtenidos toman en cuenta tanto las coordenadas geográficas de las estaciones climatológicas como su elevación; la respuesta de los modelos se comparó contra los resultados obtenidos con ayuda de regresiones lineales múltiples, presentando un mejor desempeño programación genética. Adicionalmente, se construyeron mapas de isoyetas para comparar las formas espaciales entre los datos de precipitación medidos y calculados en la cuenca del Río Cutzamala para una tormenta máxima histórica registrada en el año 2006, observándose concordancia en los resultados en el caso de precipitaciones mayores de 23 mm. La programación genética representa una herramienta de utilidad práctica para aproximar modelos matemáticos de variables aplicadas en problemas de ingeniería y se pueden obtener nuevos modelos en distintas cuencas al aplicar estos algoritmos.

----------Palabras clave: precipitación diaria, programación genética, modelos de interpolación, isoyetas, coordenadas geográficas

Introduction

Precipitation records in many basins of Mexico have a lot of missing data: different times and different existing databases in government agencies (National Water Commission, National Weather Service, the Federal Electricity Commission). There are various techniques of interpolation of precipitation trying to solve this problem such as the method of inverse distance weighted [1], Kriging and Co Kriging methods [2, 3], multifractal analysis and neural networks [4, 5], among others; lots of procedures for fitting curves have been applied in the hydrological field [6-9]. In recent years, evolutionary computation algorithms have been used in hydrology and hydraulics problems to approximate some processes such as the rainfall-runoff relationship, parameters determination on distribution functions, estimation of bivariate distribution functions or approximate equations for estimating the channel temperature from other measured climatological data [8, 10-13].

We use Genetic Programming (GP) algorithm in this paper to approximate the total rainfall in a basin, having information such as geographical coordinates and altitude. We obtained our models with total precipitation data from stations that recorded the storm of July 29th, 2006 in Cutzamala River Basin; In a first trial we only used 7 from 15 stations in order to find out the GP interpolation model capacity; then, we used 15 stations in the rest of the test. All models were tested in an artificially removed station. Additionally, isohyets from real and calculated data with GP were drawn with Kriging method to compare their shapes. A Co Kriging type method was also used in the removed station. The results obtained with the last GP model were encouraging, as described herein.

Methods

Genetic Programming

Genetic programming algorithm (GP) [14-16] is a subclass of genetic algorithms; they are inspired in Darwin's Natural Selection Theory, where the best individuals survive from each generation and the remaining disappear. The GP algorithm

traditionally consists in randomly generating an initial population of N tree individuals that, in this case, represent mathematical models, formed from a set of primitives (functions and variables) according to the problem to be solved (see Figure 1). Subsequently, as in the case of genetic algorithms [17], each individual is tested in an objective function to evaluate its performance; the best ones are selected (with random methods such as Roulette Wheel Method, Stochastic Universal Method, Tournament). These selected individuals are recombined and mutated in order to generate a new population of N individuals, which becomes the next generation and the process is repeated until a tolerance or a number of generations is reached.

Result=(x+y)-(cos(2x))
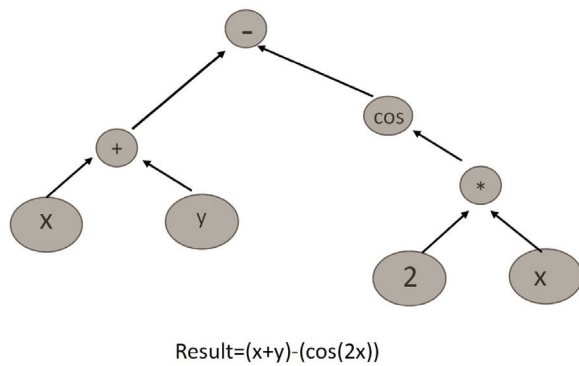
Figure 1 Mathematical model example with Genetic programming

In this study we considered arithmetic operators $\{+, -, *, /\}$, transcendent operators $\{exp, sen, cos\}$ and variables $\{x, y, z\}$ representing the values of the longitude, latitude in UTM and altitude in thousands of meters, respectively, to obtain a complete precipitation function $hp\,(x, y, z)$ in mm. A Genetic programming algorithm coded in Matlab [18] and developed in the Research Institute of Applied Mathematics and Systems, National Autonomous University of Mexico was used in the tests performed.

In the first test, we considered 200 individuals and 35 nodes while in the last test we considered 600 individuals and 32 nodes. In all cases, 5000 generations or iterations were set to end the process.

The parameters crossover (recombination) and mutation probabilities of the GP algorithm were set as 90% and 10%, respectively. The considered objective function was defined as the minimization of the mean square error between the function of total rainfall that historically occurred in a selected date and the function of precipitation calculated with genetic programming algorithm (Eq. 1):

$$Z = min\frac{1}{n}\sum_{i=1}^{n}(hpi_i(x,y,z) - \hat{h}p_i(x,y,z))^2 \quad (1)$$

Study site

Cutzamala River Basin

Cutzamala River Basin is located in central Mexico; it begins at the birth of Zitacuaro River and the downstream runoff volcanic axis of western slopes of the mountains of Angangueo, Zitacuaro River, which originates in the State of Michoacan, and Tilostoc River in Mexico State, from the ranging station of El Gallo located at the geographical coordinates 100º40'52' ' west longitude and 18º41'15 " N latitude.

Cutzamala River watershed (Figure 2), is a sub basin of the 18th Hydrologic Region Balsas; it has a contribution area of 10,619.14 km², and is bounded by the following regions and watersheds: the North by the 12th Hydrologic Region Lerma-Santiago; South by Middle Balsas River watershed; West by River watershed Tacámbaro; and east by the watershed Amacuzac River.
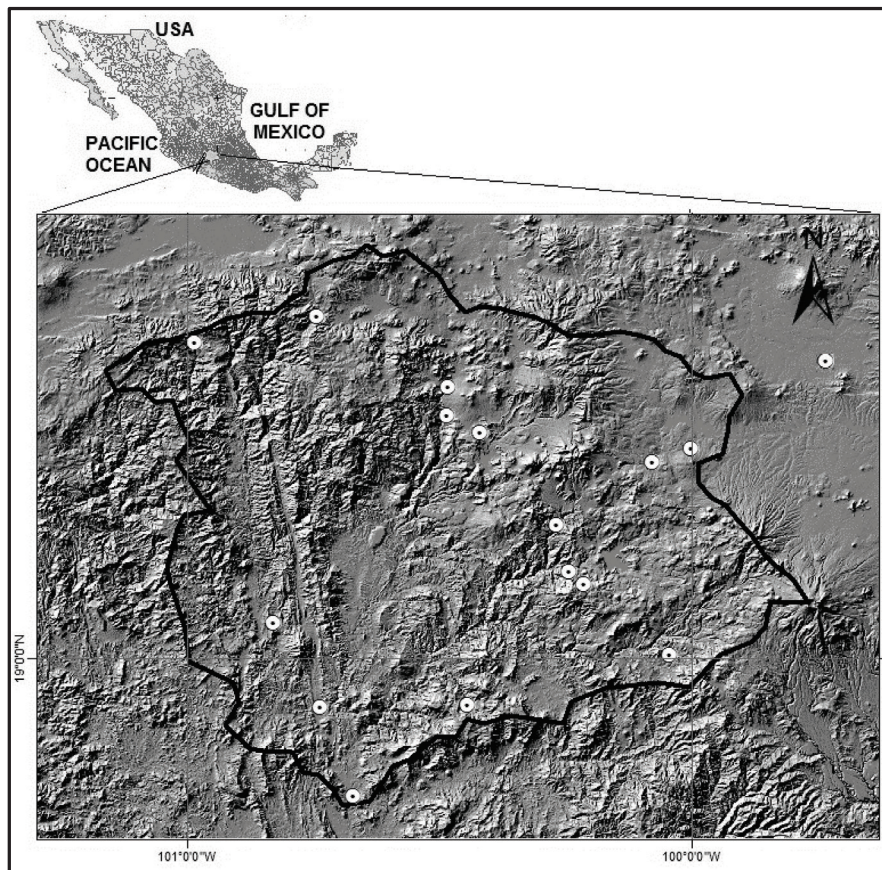
Figure 2 Cutzamala River Basin and weather stations

Input data

In the 18th Hydrologic Region Balsas, the annual rainfall volume reaches the 108 370 mm³, with an annual average rainfall of 927 mm, varying between 873 mm in the Lower Balsas and 1 019 mm in the Medium Balsas. The highest rainfall is in Southern Mother Mountain in Oaxaca and Guerrero with 2 000 mm, and the lower is in Apatzingan Valley in the "Tierra Caliente michoacana" with 600 mm. The annual average evaporation reaches 1 750 mm. The rainfalls mainly take place between June and September. The main climate in the region is semi-warm and sub-wet with an annual average temperature of 22° [19].

The historical storm of July 29th, 2006, which is one of the greatest historical storms throughout the record, was selected to perform this study. Rainfall gauge stations with pluviometer, which recorded the height of precipitation (hp) in mm, were chosen on that date; longitude and latitude coordinates (x, y) in UTM (in thousand meters) and altitude (z) in thousands of meters were used to obtain the model. Additionally, the assumption that the weather station 15046 Colorines Dam was not able to measure precipitation was considered. (The station was randomly selected from the cluster of stations allocated approximately in the middle of the basin (Figure 3). This station was not considered for the model (Table 1) and was subsequently used to verify the response model for interpolating missing data.
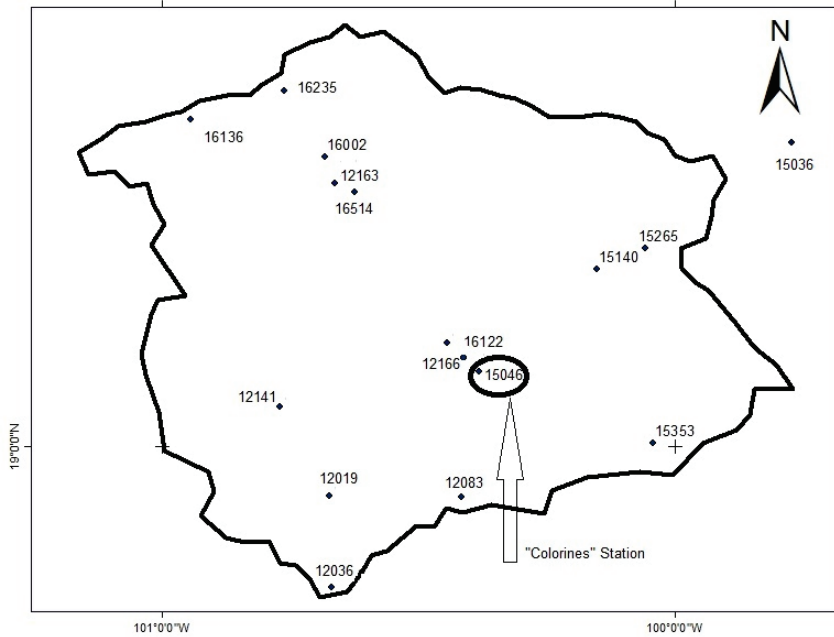
Figure 3 Location of the "Colorines" Climatologic station

Table 1 Stations considered for the model

Key	Name	Mexico State
12083	Tehuehuetla, S. M.Totolapa	Guerrero
12166	San Miguel Totolapan,	Guerrero
12141	Tlapehuala, Tlapehuala	Guerrero
12019	Iguala	Guerrero
12163	Cutzamala de Pinzon	Guerrero
12036	El Gallo, Cutzamala P	Guerrero
15036	Ixtlahuaca, Ixtlahuaca	Mexico S
15353	Buenavista	Mexico S
16122	Susupuato Guerrero,	Michoacan
15140	P.Chilesdo, V.De Allende	Mexico S
15265	Camp. Berros, San Jose	Michoacan
16514	Jaripeo, Tip	Michoacan
16002	Agostitlan, Cd. Hidalgo	Michoacan
16136	Tzitzio, Tzitzio	Michoacan
16235	Huajumbaro, Cd. Hidalgo	Michoacan

Results and discussion

In order to find out if with few data the model can be capable to interpolate surrounding data, we applied genetic programming algorithm to the first 7 data from a total of 15 measured precipitation records which appear in Table 2 (in this part we also did not consider the Colorines dam station 15046): Thus, the following mathematical model was determined (Eq. 2):

$$\hat{hp} = \frac{x}{\left[\frac{x}{senx} - exp(exp(z))\right] * exp\left[\frac{cos(0,378807)}{0,200149}\right]} - \frac{1}{sen(x)}$$

$$\left[\frac{sen \; exp(x) - (\frac{z - sen(y)}{-0,479954})}{-0,170958}\right] - exp(z) \qquad (2)$$

The objective function for this model gave us a value of 130.88, this result does not seem to be good enough because it is about the mean square error, and it was necessary to review the correlation coefficient between the measured and the calculated data to get a conclusion about

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this result. This model was then used to obtain calculated rainfall data in the 15 stations using longitude, latitude and elevation information. This information was obtained for stations that recorded data for total rainfall for the selected storm (Table 2).

Table 2 Results of the application of the GP model Eq. (2). Storm July 29th, 2006 Cutzamala Basin

Code	x (10^3m)	y (10^3m)	z (10^3m)	hp (mm)	hp$_{GP}$ (mm)	Error (Mm)
12083	351.6	1970.4	1.25	20	22.3	-2.34
12166	353.7	2009.2	0.28	104.7	102.7	1.95
12141	341.4	2016.6	0.27	50	53.8	-3.82
12019	323.8	2026.0	0.25	5	0.8	4.14
12163	338.1	2042.5	0.26	0	1.7	-1.79
12036	324.2	2072.1	0.4	23.2	20.9	2.22
15036	415.7	2091.8	2.17	24.7	24.3	0.35
15353	391.2	2101.1	2.57	2.3	5.6	-3.39
16122	351.0	2121.6	1.56	23	25.7	-2.71
15140	379.0	2139.6	2.39	19.3	22.3	-3.09
15265	465.3	2138.3	2.15	20	0.5	19.4
16514	342.5	2151.3	1.3	14.6	5.6	8.97
16002	330.3	2160.6	2.38	7	23.6	-16.6
16136	298.9	2166.4	1.85	4	31.3	-27.3
16235	318.3	2175.5	2.28	20	0.1	19.9

It was observed the model reported similar results in about 9 of the 15 selected stations, with a minimum absolute error of 0.35 mm in the calculated hp for 15036 Ixtlahuaca station and a relative error of 1.42%; while at station 16136 Tzitzio, the maximum difference of 27.3 mm was obtained with a huge relative error of 682.5% .

The measured and calculated total rainfall data were drawn and compared with an identity function (Figure 4), showing the close grouping to the identity nearly 60% of the data with a determination coefficient of 0.8084. Additionally, the determination coefficient estimated to account for the nonlinear model was obtained, this coefficient gave 0.7916 and was obtained with the variance (*var*) of the measured data (var_{hp}) and the variance of the error in the estimation of rainfall ($var_{hperror}$). That is $C_{det} = (var_{hp} - var_{hperror}) / var_{hp}$.

Subsequently, we used our model to estimate the value of the total precipitation at 15046 Colorines Dam station with coordinates x= 375.5569489 10^3 m and y=2112.312711 10^3 m and a height z= 1.68 10^3 m, determining a value of hp=6.60 mm. Moreover, the measured data at that station was 8 mm, indicating the absolute error was 1.4 mm and the relative 17.5%. This data was drawn in the Figure 4 to locate its position regarding the identity function.

Additionally, isohyets were drawn using ARCMAP© and Kriging interpolation, taking into account measured and calculated data with the GP model (Figures 5 and 6).
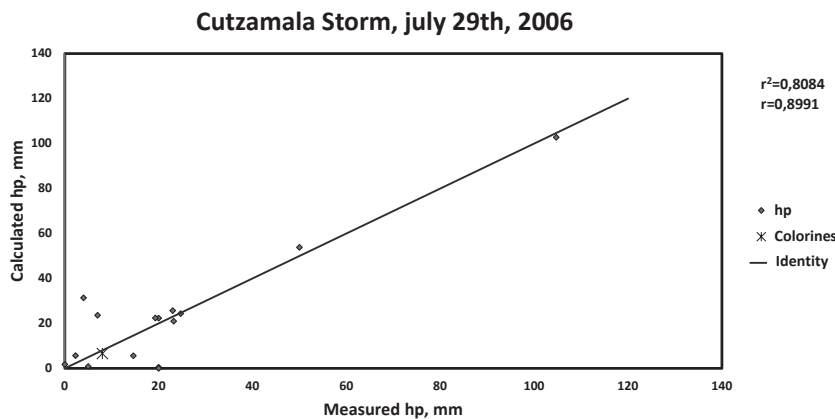
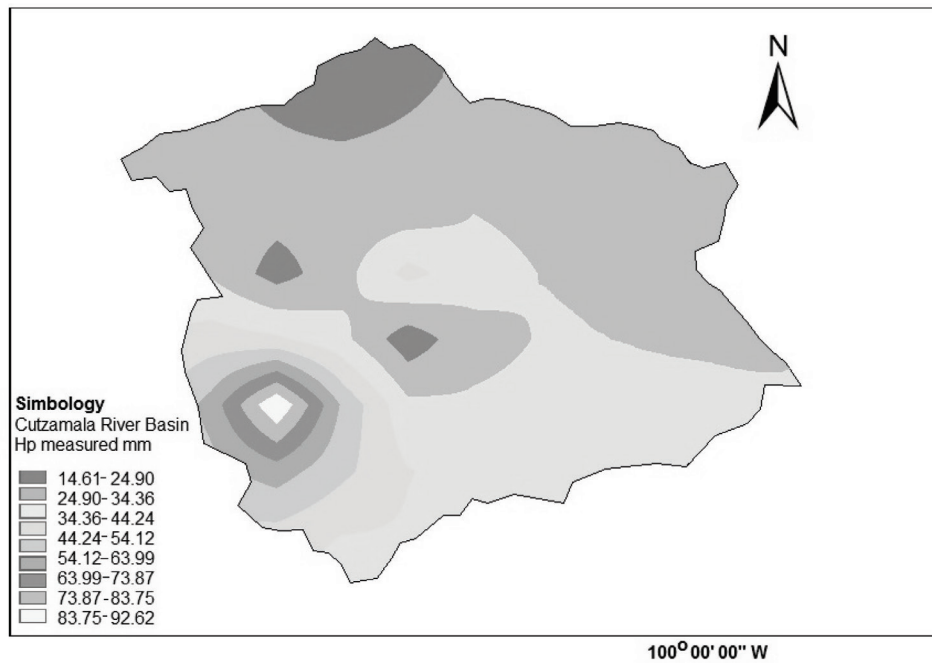**Figure 4** Comparison between measured and calculated data and interpolated data. From Eq. (2)

Figure 5 Measured Isohyets data. July 29*th* Storm, 2006
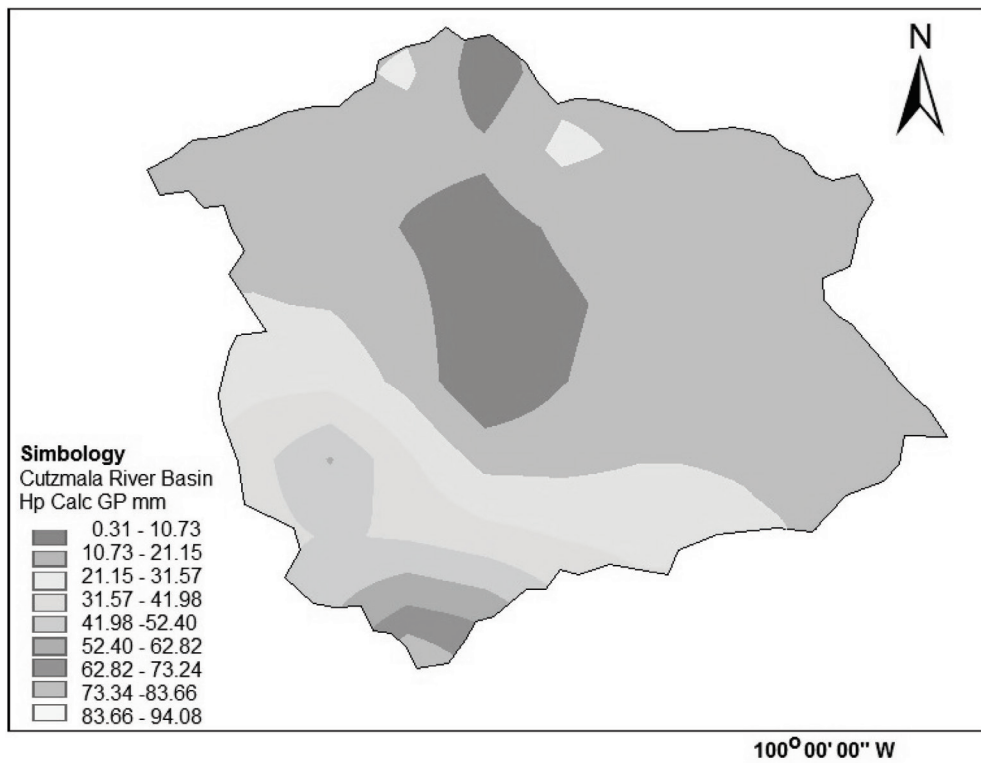

Figure 6 Calculated *hp* Isohyets obtained with GP Eq. (2). July 29*th* Storm, 2006

We can see that Figures 5 and 6 have similar shapes only in their Isohyets near to 20 mm (Figure 4), while for other values the model has difficulties in reproducing the historical behavior.

Then, trying to improve the results by considering more data, a new GP model using the 15 stations was obtained, and we did not also consider the Colorines dam station. In this case the GP model was as shown in the Eq. (3):

$$\widehat{hp} = exp\{exp\left[sen\left(sen\left(sen\left(\frac{z}{0,713368}-y\right)\right)\right)+\right.$$

$$cos\left[y-[exp(sen(sen(1,538638-y)))-0,717702]\right]\right]$$

$$- cos(0,611312\, cos(z))\} \tag{3}$$

The mean square error in this model was 512.47, and it was observed that 7 points gave an absolute error lower than 10 mm compared to measured (Table 3). Table 3 shows that the least absolute error was 0.58 mm for the precipitation measuring 23 mm and a relative error of 2.52%, whereas the maximum absolute error was 66.1 mm, for the precipitation of 50 mm with a relative error of 132.2%. Based on this model, the rainfall was calculated for Colorines dam station and the measured and calculated data were plotted against the identity function (Figure 7). In this case, the calculated value for Colorines dam station was 12.93 mm against the 8 mm had occurred

historically, it represents an absolute error of 4.93 mm when using Eq. (3) with a relative error of 61.63%. It means Eq. (3) gave a larger error than one obtained with Eq. (2); that is attributed to the dispersion values of the 15 data against the dispersion values of the 7 data.

Table 3 Results of the application of the GP model Eq. (3). July 29th Storm, in 2006 Cutzamala Basin

| key	x (10³ m)	y (10³ m)	z (10³ m)	Hp (mm)	h_{PGP} (mm)	error (mm)
12083	351.6	1970.4	1.25	20	6.61	13.39
12166	353.7	2009.2	0.28	104.7	97.7	7
12141	341.4	2016.6	0.27	50	116.17	-66.17
12019	323.8	2026.0	0.25	5	8.69	-3.69
12163	338.1	2042 5	0.26	0	6.83	-6.83
12036	324.2	2072.1	0.4	23.2	29.54	-6.34
15371	415.7	2091.8	2.17	24.7	11.17	13.53
15353	391.2	2101.1	2.57	2.3	6.01	-3.71
16122	351.0	2121.6	1.56	23	23.58	-0.58
15140	379	2139.6	2.39	19.3	62.07	-42.77
15265	465.34	2138.3	2.15	20	34.81	-14.81
16514	342.5	2151.3	1.3	14.6	35.7	-21.1
16002	330.3	2160.6	2.38	7	17.38	-10.38
16136	298.9	2166.4	1.85	4	16.1	-12.1
16235	318.3	2175.5	2.28	20	14.1	5.9
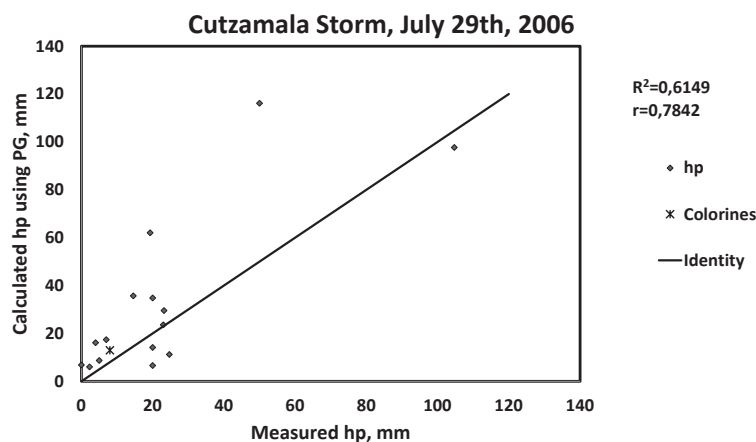
Figure 7 Comparison between measured and calculated data and interpolated data. Eq. (3)

Taking the 15 stations, the multiple linear regression model obtained with supplements Excel spreadsheet© took the following form (Eq. 4):

$$\widehat{hp} = 0.136885x - 0.061247y -$$

$$10.452978z + 116.972596 \qquad (4)$$

The determination coefficient between measured and calculated rainfall data was 0.4734 (Figure 8), which is lower than the result obtained with the GP model.
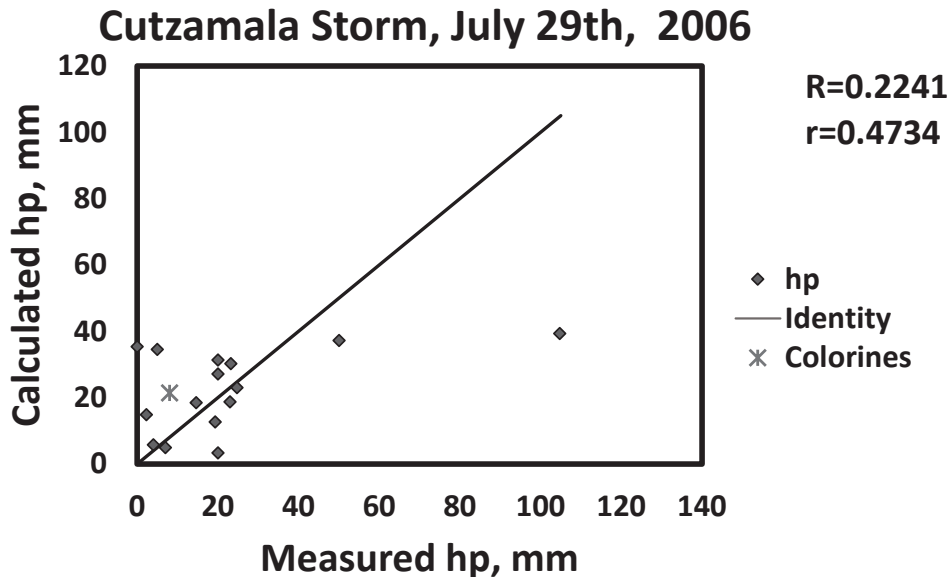
Figure 8 Comparison between measured and calculated data and interpolated data. Eq. (4)

The estimated rainfall for the Colorines dam station by applying the linear regression model of Eq. (4) was 21.45 mm against the real value of 8 mm; that represents an absolute error of 13.45 mm and a 168% relative error. In the case of the interpolation of a missing data, Genetic programming model of Eq. (2) was the optimal relative error of 17.5%.

A last model was obtained in an attempt to improve the response patterns of genetic programming using the 15 considered stations, 60 individuals, 32 nodes; the function set included both algebraic (+,-,*,/) and transcendental (cos) operators and a terminal set of constants randomly created (*R)*, and the variables x, y, z.

The new obtained model was (Eq. 5):

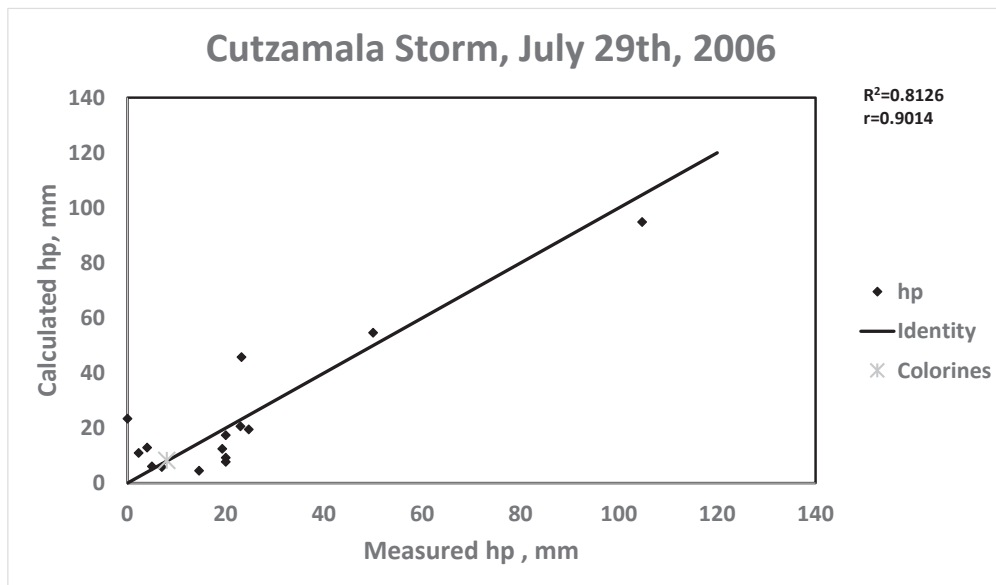$$\widehat{hp} = A + \left(\frac{B}{C}\right) \qquad (5)$$

Where:

$$A = 2\ cos(x)$$

$$B = 1.0127 + z + cos(xy) +$$

$$cos(-0.16484y^2) + cos(cos(-0.8827x)$$

$$C = 0.14648z$$

Based on this latest model, the mean square error of the objective function was 119.15; in this case the determination coefficient was 0.8126, that is a correlation coefficient of 0.9014 (Figure 9). Table 4 shows the measured and the calculated values obtained with Eq. (5).

Figure 9 Comparison between measured and calculated data and interpolated data. Eq. (5)

Table 4 Results of the application of the GP model Eq. (5). July 29th Storm, in 2006 Cutzamala Basin

key	hp	hp$_{GP}$	error
	mm	mm	mm
12083	20	17.43	2.57
12166	104.7	94.84	9.86
12141	50	54.62	-4.62
12019	5	6.07	-1.07
12163	0	23.38	-23.38
12036	23.2	45.75	-22.55
15371	24.7	19.46	5.24
15353	2.3	10.93	-8.63
16122	23	20.65	2.35
15140	19.3	12.44	6.86
15265	20	9.19	10.81
16514	14.6	4.48	10.12
16002	7	5.83	1.17
16136	4	12.90	-8.90
16235	20	7.69	12.31

The interpolated value for Colorines dam station model Eq. (5) was 8.22 mm, representing an absolute error of 0.22 mm and a relative error in percent of 2.75% (Figure 9). This result leads to consider Eq. (5) as the best result of all the tests performed for interpolation purposes and it was the second best result in validating data.

The isohyets corresponding to results of Eq. (5) appear in Figure 10. In this case the model reproduces the shapes of isohyets.
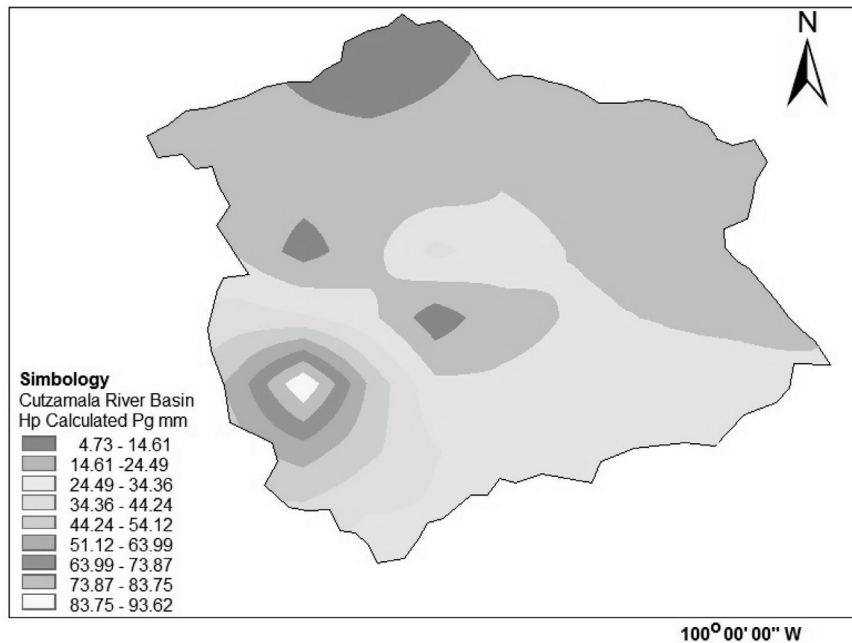
Finally, a Co Kriging method used in scientific field was applied to compare all the results in case of Colorines dam station; the applied optimization model was (Eq. 6):

$$FO=Min[(Ehp^2)+(\textstyle\sum_{i=1}^{n}a_i-1)^2+(\textstyle\sum_{j=1}^{n}b_j-0)^2] \quad (6)$$

Where:

$$Ehp = hp - \widehat{hp} \quad (7)$$

$$\widehat{hp} = \textstyle\sum_{i=1}^{n} a_i hp_i + \textstyle\sum_{j=1}^{n} b_j hp_j \quad (8)$$

Figure 10 Calculated *hp* Isohyets obtained with GP Eq. (5). July 29*th* Storm, 2006

The parameters obtained in Excel© with Solver© were as shown in Table 5.

Table 5 Coefficients of co Kriging (Eq. 8).

Coefficient	Value
a_1	0.054090977
a_2	0.000002756
a_3	0.087969864
a_4	0.312094132
a_5	0.511124398
a_6	0.023104993
a_7	0.011119652
b_1	0.000000000
b_2	0.000000000
b_3	0.000000000
b_4	0.000000000
b_5	0.000000000
b_6	0.000000000
b_7	0.000000000
b_8	0.007412067

The value for Colorines dam station obtained with Eq. (8) was $\widehat{hp}$ = 8.0000 mm; it means the methodology used was the best, for this particular case.

Conclusions

Using genetic programming to interpolate rainfall data to a historical maximum storm recorded at Cutzamala River Basin, was determined with the model of Eq. (2), where an adjustment of about 60% of the data considered was achieved; its obtained isohyets were similar to those recorded historically, mainly for precipitation data from 23 mm. With the model of Eq. (2) the smallest relative interpolation error was obtained for Colorines dam station.

When all the data were considerate for the GP model (eq 3) between linear correlations and measured calculated data decreased. This was attributed to the spatial distribution of the stations in the xy plane, in addition to the space distribution of rainfall respect to altitude. Using the GP model (eq 3) also gave higher correlations than those used with the simple regression model; likewise, it also produced the lowest relative error within interpolated data taken on the Colorines station, but this model was no able to overcome the GP model of Eq. 3.

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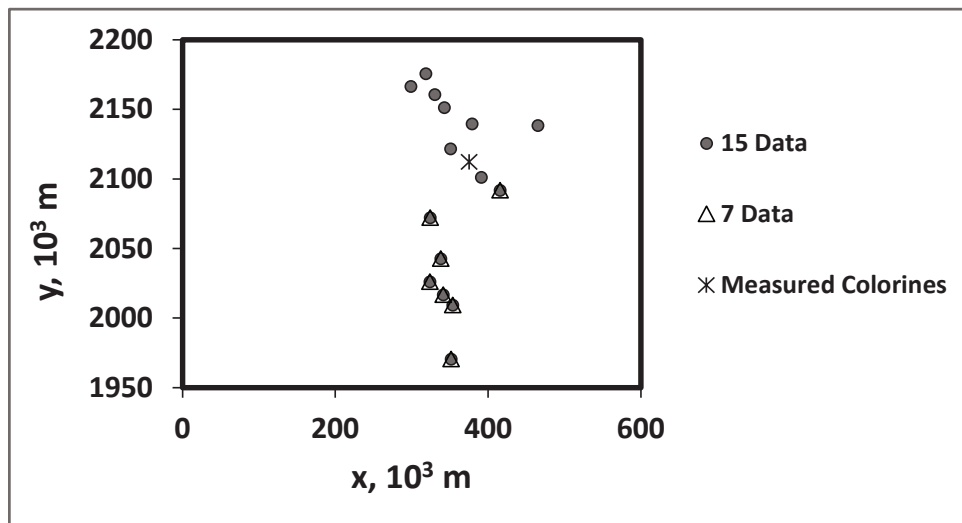
Figure 11 Location stations in the *xy* plane
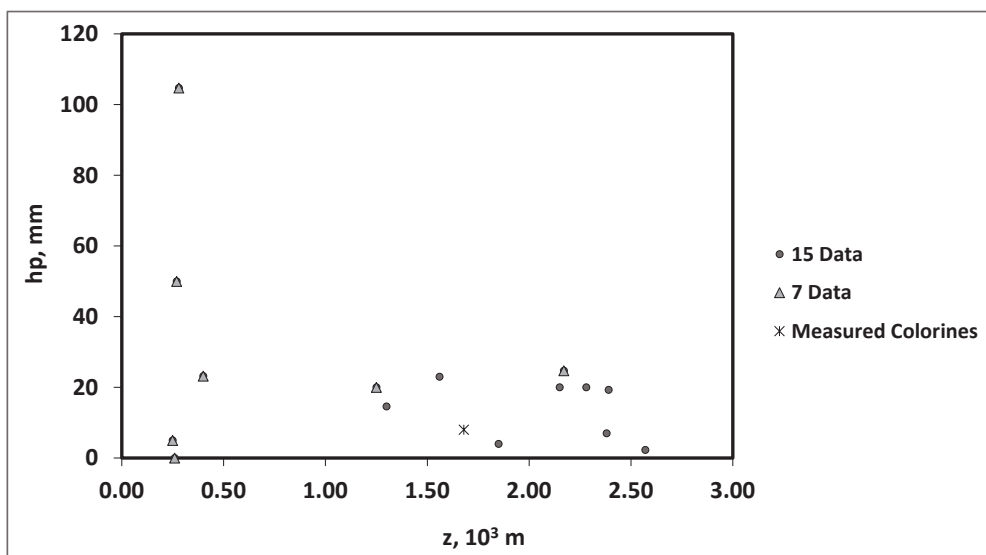
Figure 12 Relationship between precipitation-height

Finally, the GP model of Eq. (5) which takes into account the 15 selected stations gave higher correlations than those obtained with the other GP and multiple linear correlation models and also gave the second best interpolation result for Colorines dam station with a relative error of only 2.75% (because the co Kriging method gave the best interpolation in this analyzed event). Thus, in this respect, the GP model of Eq. (5) can be

considered of practical use. And with this model the best Isohyets were obtained as well.

It is very desirable for future developments to use average historical precipitation values in the analyzed sites for generating GP interpolation models, as see the convenience of analyzing a larger number of rainfall events in order to improve the results.

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